CHAPTER ELEVEN

Cuboids, Cylinders and Circles

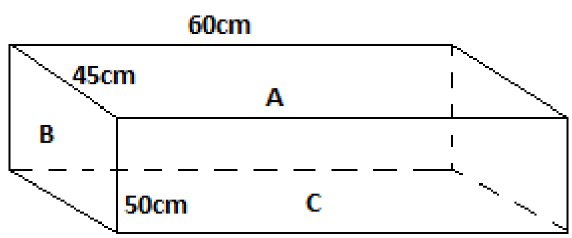
(Q1) A rectangular water tank has a length of 60cm, width 45cm and height 50cm.

Find (i) the total surface area of the tank when closed.

(ii) the volume of the tank.

(iii) the height of the water in the tank, if the tank contains 81,000cm^3 of water.

Soln:



In the diagram, three surface areas can be seen i.e. A, B and C.

Each of these has an equal counterpart of the same dimensions and this is directly opposite it.

<u>Surface A:</u>

L = 60cm and B = 45cm.

Area = L x B = 60 x 45 = 2700cm^2.

Area of the opposite counterpart of surface A is also = 2700cm^2.

Area of surface A and its opposite counterpart = 2700 + 2700 = 5400cm^2.

<u>Surface B:</u>

L = 45cm and B = 50cm.

Area = L x B = 45 x 50 = 2250cm^2.

Area of the opposite counterpart of surface B is also = 2250cm^2.

Area of surface B and its opposite counterpart = 2250 + 2250 = 4500cm^2.

<u>Surface C:</u>

L = 60cm and B = 50cm.

Area = L x B = 60 x 50 = 3000cm^2

Area of the opposite counterpart of surface C is also = 3000cm^2

Area of surface C and its opposite counterpart

$= 3000 + 3000 = 6000cm^2$

Since the tank is closed, then all its six surfaces are available.

Total area of the closed tank = area of surface A and its opposite counterpart + area of surface

B and its opposite counterpart + the area of surface C and its opposite counterpart = 5400 + 4500 + 6000 = $15900cm^2$.

(ii) The volume of the tank

= L x B x H = 60 x 45 x 50

= 135, 000 cm^3

(III) V = L x B x H,

Dividing through using L x B=> $\frac{V}{\text{L X B}} = \frac{L \text{ x B x H}}{\text{L X B}}$, => H = $\frac{V}{\text{L X B}}$ => $H = \frac{81000}{60 \text{ X } 45}$

= 30.∴ The height of water in the tank, if it contains $81{,}000cm^3$ of water is 30cm.

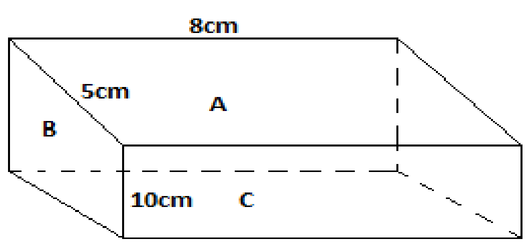
(Q2) A box has length 8.0cm, width 5.0cm and height 10.0cm. Find the:

(i) total surface area of the box.
(ii) the volume of the box.

N/B: 8.0cm = 8cm, 5.0cm = 5cm and 10.0cm = 10cm.

Soln:

(i)



Surface A:
L = 8cm and B = 5cm.
Area = L x B = 8 x 5 = $40cm^2$.
Area of surface A and its opposite counterpart = 40 + 40 = $80cm^2$.

Surface B:
L = 5cm and B = 10cm
Area = L x B = 5 x 10 = $50cm^2$
Area of surface B and its opposite counterpart = 50 + 50 = $100cm^2$.

Surface C:
L = 8cm and B = 10cm
Area = L x B = 8 x 10 = $80cm^2$
Area of surface C and its opposite counterpart = 80 + 80 = $160cm^2$
Total surface area of the box
= Area of surface A and its opposite counterpart + area of surface B and its opposite counterpart+ area of surface C and its opposite counterpart = 80 + 100 + 160
= $240cm^2$.

(ii) L = 8cm, B = 5cm and H = 10cm.
Volume of the box = L x B x H
= 8 x 5 x 10 = $400cm^3$.

(Q3) A water tank in the form of a cuboid of height 22cm and a rectangular base of length 7cm and width 5cm is filled with water. The water is then poured into a cylindrical container of diameter 14cm. Calculate the height of water in the cylindrical container [Take $\pi = \frac{22}{7}$].

Soln:

Volume of water in the tank

= L x B x H = 22 x 7 x 5 = $770cm^3$.

Since the water in the tank was poured into the cylindrical container, then the volume of water in the tank will be equal to that in the cylindrical container.

Since the volume of water in the cylindrical container = $\pi r^2 h$, where h is the height of water in the cylindrical container, and r = its radius = $\frac{14}{2} = 7$ (since the diameter = 14cm), then $\pi r^2 h$= 770

=> $\frac{22}{7}$ x 7^2 x h = 770,

=> $\frac{22}{7}$ x 7 x 7 x h = 770,

=> 22 x 7 x h = 770,

=> $\frac{22 \text{ x } 7 \text{ x h}}{22 \text{ x } 7} = \frac{770}{22 \text{ x } 7}$

i.e. divide through using 22 x 7,

=> h = 5.

∴ The height of the water in the cylindrical container = 5cm.

(Q4) A rectangular tank of length 22cm, width 9cm and height 16cm is filled with water. The water was then poured into a cylindrical container of radius 6cm. Calculate the

(i) volume of the rectangular tank.

(ii) the depth of water in the cylindrical container [Take $\pi = \frac{22}{7}$].

Soln:

(i) The volume of water in the rectangular tank = L x B x H
= 22 x 9 x 16 = 3168cm^3.

(ii) The volume of water within the cylindrical container = $\pi r^2 h = \frac{22}{7} \text{ x } 6^2 \text{ x h}$
= $\frac{22}{7}$ x 6 x 6 x h, where h = the height or the depth of water within the cylindrical container.
Since the volume of water within the cylindrical container = the volume of water within the rectangular tank, then $\frac{22}{7}$ x 6 x 6 x h = 3168, => $\frac{22 \text{ x } 6 \text{ x } 6 \text{ x h}}{7} = 3168$.

By cross multiplication =>

22 x 6 x 6 x h = 7 x 3168.

Dividing through using 22 x 6 x 6

=> $\frac{22 \text{ x } 6 \text{ x} 6 \text{ x h}}{22 \text{ x } 6 \text{ x } 6} = \frac{7 \text{ x } 3168}{22 \text{ x } 6 \text{ x } 6}$

=> h = $\frac{7 \text{ x } 3168}{22 \text{ x } 6 \text{ x } 6} = 28$.

Therefore the depth of water within the cylindrical container = 28cm.

N/B:

(I) If a cylinder is closed at both ends, then it has both the top circular surface, as well as the bottom circular surface.
Its total surface area = the bottom circular surface area + the top circular surface area + the curved surface area
= $\pi r^2 + \pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$.

(II) If he cylinder is closed at one end, then it contains only one of the circular surfaces.

i.e. either the bottom or the top circular surface.

The total surface area of such a cylinder = either the top or the bottom circular surface area + the curved surface area = $\pi r^2 + 2\pi rh$.

But in each of these two cases mentioned, the volume of the cylinder = $\pi r^2 h$.